Progress in Time-Domain (FDTD) Electromagnetic Modeling

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Acknowledgement

Prof. Wen-xun Zhang was one of the examining committee members for my Master thesis in 1982. Since then, he has been my mentor and colleague who shared his knowledge, experience, enthusiasm and candid opinions with me whenever we met. I am lucky and grateful to have him as a teacher and a senior colleague.

I would also like to thank the conference organizers for giving this opportunity to present in this special session honoring Prof. Wen-xun Zhang’s long career, dedication and achievements in our profession. Unfortunately, due to my unexpected medical appointments with my doctor in Canada, I am unable to make it to this session; instead, I would also my young colleague, Dr. Huapeng Zhao, one of the next generation professionals, to make a technical presentation for me.

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Abstract

Electromagnetics forms the foundation for electrical and computer engineering(ECE). Therefore, the first-principle solutions to ECE hardware-related problems and issues can be obtained by solving electromagnetic fields which are governed by Maxwell’s equations. Since only few analytical solutions are readily available for structures of regular shapes, numerical methods must be employed; many of them have been developed so far including the popular finite-different time-domain method (FDTD) and finite-element method (FEM). In the past 30 years, thanks to the tremendous increase of computer power, many software packages, based on different numerical methods, have been developed and made available in markets. Still, the challenges in computational memory and efficiency remain, in particular for Multiscale and Multiphysics modeling and simulations.

In this presentation, we provide the summary of the three recent developments pertaining to the time-domain methods for electromagnetic modeling; they are the analytical FDTD solutions, a practical time-reversal method and a unifying node-based computational platform. The first development gives an in-depth look of the FDTD solution structures in terms of numerical eigen-mode expansions; it presents the ways to achieve multi-scale and multi-grid modeling. The second development allows the effective source locating and possible structure synthesis that avoids false solutions and can be carried out in a large computational domain. The third development shows the possibility of having most of numerical methods under a single mathematical framework and then hybridizing them. These three developments present potential new horizons for efficient and effective numerical modeling of Multi-scale and Multiphysics problems. Numerical examples are given to illustrate the points.

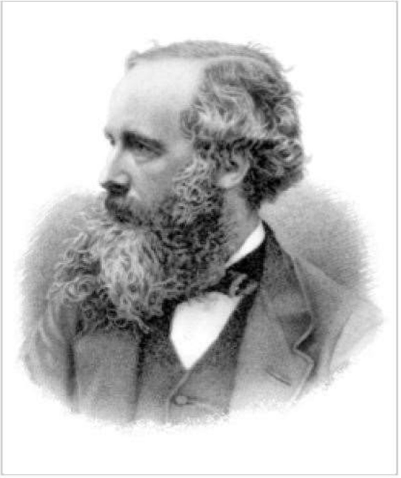
Index terms: finite-difference time-domain (FDTD) method, finite-element method (FEM), time-domain modeling, eigen modes, multiscale, Multiphysics, time reversal, source reconstructions, node-based mesheless methods, unification, hybridization.

Outline

- Introduction
- Analytical FDTD Solution
- A Practical Time-Reversal Scheme
- A Unifying Computational Platform
- Conclusion

Introduction

- Electromagnetic model: Maxwell’s equations of 26 equations by James Clark Maxwell in 1864; they quantify and summarize all the previous and experimental results and describe the behaviors of electromagnetic fields.


James Clerk Maxwell.

- Modern Maxwell’s Equations: simplification of the 26 equations to the 4 equations we see today by Oliver Heaviside in 1885 with introduction of flux and vector operations.

Oliver Heaviside.

Introduction

- Electromagnetic Fields: foundations of electrical and computer engineering → electric power, cars, radios, cellular phones, WiFi, computers etc.

- Electrical and Computer Engineering Problems: solutions of Maxwell's equations:

$$\frac{\partial \vec{\mathbf{D}}(x,y,z,t)}{\partial t} = \nabla \times \vec{\mathbf{H}}(x,y,z,t) - \vec{\mathbf{J}}(x,y,z,t)$$

$$\frac{\partial \vec{\mathbf{B}}(x,y,z,t)}{\partial t} = \nabla \times \vec{\mathbf{E}}(x,y,z,t)$$

$$\vec{\mathbf{D}} = [\varepsilon]\vec{\mathbf{E}}; \qquad \vec{\mathbf{B}} = [\mu]\vec{\mathbf{H}}$$

Introduction

- Solutions of Maxwell's equations:
  - Exact analytical solutions:
    only possible for a few regular geometry of structures.
  - Approximate solutions:
    - Circuit theory and lump element models (approximated models of Maxwell's equations): workable at low frequencies (i.e. structure size 10 times smaller than wavelength): Ohm's Law, Kirchhoff's Voltage & Current Laws.
    - Numerical methods:
      - Time-domain methods;
      - Frequency-domain method.

Introduction

- **Time-domain methods: Three major techniques:**
  - Finite-difference time-domain (FDTD) methods:
    - Derivatives in Maxwell’s equations → finite-differences in Maxwell’s equations.
  - Finite-element time-domain (FEM-TD) methods:
    - Solution domains → discretized with triangular or tetrahedral elements;
    - solutions → expansions with linear shape functions in in each element; testing with Dirac impulse functions;
    - temporal derivatives → finite difference.
  - Method of Moments - Time-domain integral equation (TDIE) methods:
    - Maxwell’s equations → integral equations to be solved;
    - Solution domains → discretized with surface elements;
    - solutions → expansions with basis function in in each element; testing with pre-selected weighting functions;
    - temporal derivatives → finite difference in most cases.

Introduction

- **Major Software packages:**
  - XFDTD:
  - CST

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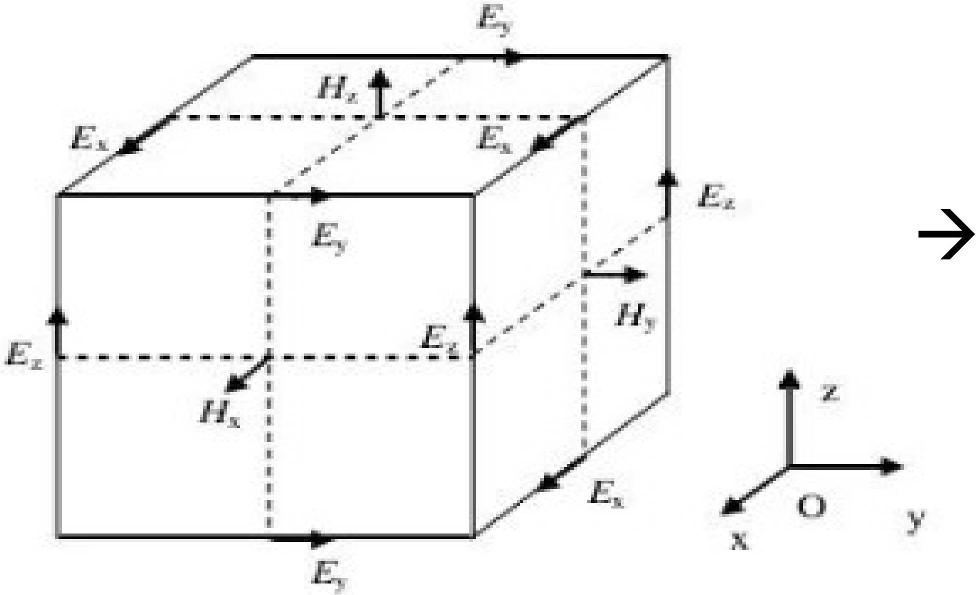
Analytical FDTD Solutions

Eigen-mode decomposition of the FDTD formulation:



→ Discretization in space → Discrete Space

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = \rho$$

Finite-difference →

$$\boxed{\begin{aligned}[H]^{n+\frac{1}{2}} &= [H]^{n-\frac{1}{2}} - \Delta t[D_E][E^n]\\ [E]^{n+1} &= [E]^n + \Delta t[D_H][H]^{n+\frac{1}{2}} - \frac{\Delta t}{\varepsilon}][J]^{n+\frac{1}{2}}\end{aligned}}$$

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Analytical FDTD Solutions

$$[H]^{n+\frac{1}{2}} = [H]^{n-\frac{1}{2}} - \Delta t[D_E][E^n]$$

$$[E]^{n+1} = [E]^n + \Delta t[D_H][H]^{n+\frac{1}{2}} - \frac{\Delta t}{\varepsilon}][J]^{n+\frac{1}{2}}$$

$$[\mathrm{M}] = [D_H][D_E]$$

Eigenvalue γ_i*;*
spatial eigenmode: $[V]_i$ *.*

FDTD solutions: superposition of spatial eigen-modes modulated with time varying amplitudes

$$[E]^n = \sum_i \alpha_i^n [V]_i$$

$$\alpha_i^{n+1} + \alpha_i^{n-1} - (2-\gamma_i)\alpha_i^n = b_i^n, \text{ for } n = 1, 2, 3, \ldots$$

Stable mode : $\gamma_i < 4$
Unstable mode : $\gamma_i \geq 4$

Stable FDTD with large time step: discard unstable modes

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Analytical FDTD Solutions

Eigen-mode decomposition of the time-domain methods:

take FDTD as an example

FDTD solutions: superposition of spatial eigen-modes modulated with time varying amplitudes

$$\boxed{E^n = \sum_i \alpha_i^n [V]_i}$$

$$\boxed{\alpha_i^{n+1} + \alpha_i^{n-1} - (2-\gamma_i)\alpha_i^n = b_i^n, \text{ for } n = 1, 2, 3, \ldots}$$

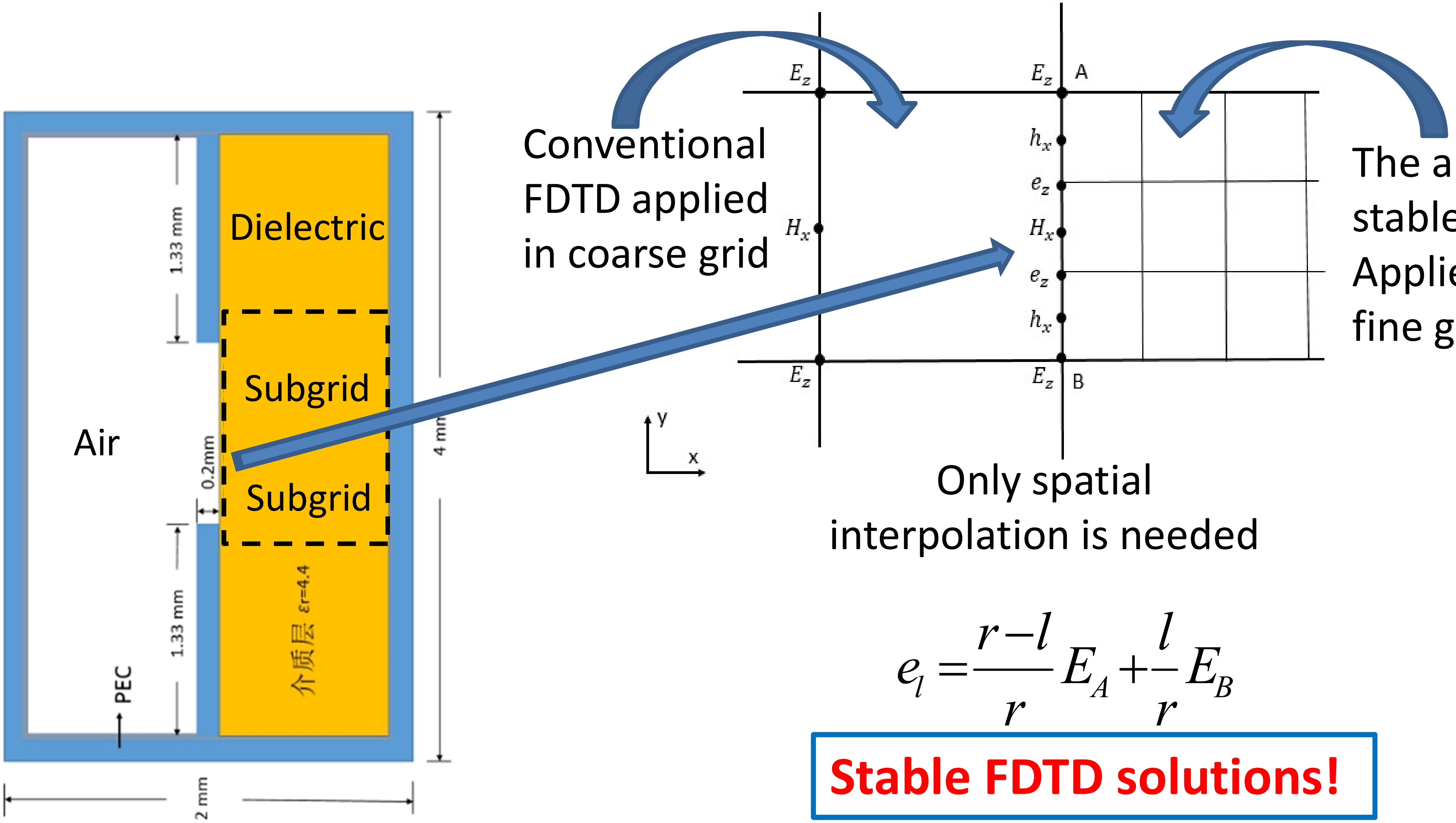
- **To find eigenmodes $[V]_i$, model-order reduction techniques maybe used since stable modes are of low-frequency**

e.g. run a few FDTD iterations

Analytical FDTD Solutions

Eigen-mode decomposition of the time-domain methods:

take FDTD as an example


Dielectric
Subgrid
Subgrid
Air
PEC
1.33 mm
0.2mm
1.33 mm
4 mm
2 mm
介质层 εr=4.4
E_z
E_z
A
h_x
e_z
H_x
H_x
e_z
h_x
E_z
E_z
B
y
x

Conventional FDTD applied in coarse grid

The above stable FDTD Applied in fine grid

Only spatial interpolation is needed

$$e_l = \frac{r-l}{r} E_A + \frac{l}{r} E_B$$

Stable FDTD solutions!

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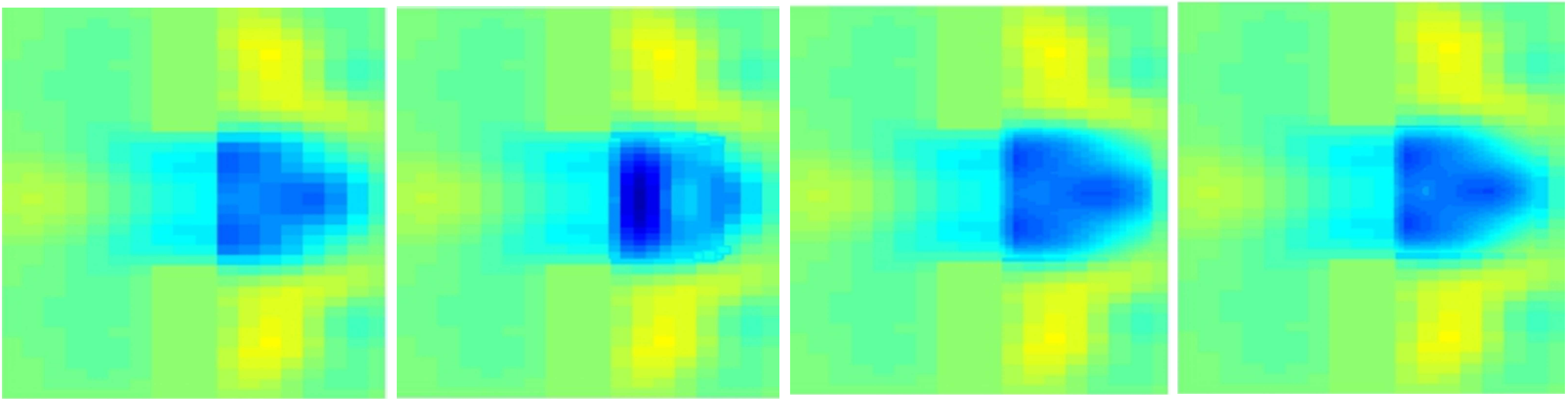
Analytical FDTD Solutions

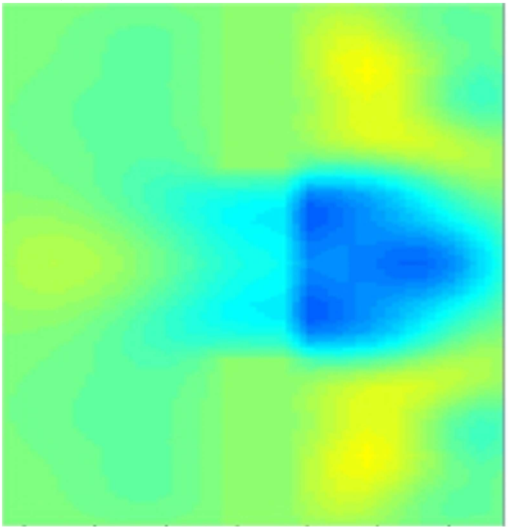
Eigen-mode decomposition of the time-domain methods:



Subgrid mesh becomes finer

M. Zhou, Z. Chen and W. Fan, “**A subgridding scheme with the unconditionally stable explicit FDTD method**,” in *Proc. IEEE Numerical Electromagnetic Modeling and Optimization (NEMO)*, July, 2016.

Reference result



Analytical FDTD Solutions

Eigen-mode decomposition of the time-domain methods:

take FDTD as an example

1. Z. Chen and Peter P. Silvester, "**Analytic solutions for the finite- difference time-domain and the transmission-line-matrix method**", *Microwave Optical Tech. Lett.*, John Wiley &Sons, vol. 7, no. 1, pp. 5-7, Jan. 1994.
2. M. Gaffar and D. Jiao, "**An explicit and unconditionally stable FDTD method for electromagnetic analysis**," *IEEE Trans. Microwave Theo. Tech.*, vol. 62, no. 11, pp. 2538-2550, Nov. 2014.
3. Z. Chen, W. Fan and S. Yang, "**Towards the wave-equation based explicit FDTD method without numerical instability**", in *Proc. IEEE Intl Conf. Comp. Electromagnetics (ICCEM), pp.* 265-267, Feb. 23-25, 2016.
4. M. Zhou, Z. Chen and W. Fan, "**A subgridding scheme with the unconditionally stable explicit FDTD method**," in *Proc. IEEE Numerical Electromagnetic Modeling and Optimization (NEMO)*, July, 2016.
5. J. Yan and D. Jiao, "**A symmetric positive semi-definite FDTD subgridding algorithm for arbitrary grid ratios with uncompromised accuracy**," in *Digests of IEEE International Microwave Symposium*, June 4-9, 2017 Honolulu.

$a_{i,n+1} + a_{i,n-1} - (2-\lambda_i)a_{i,n} = b_{i,n}$, for $n = 1, 2, 3, \ldots$

Analytical FDTD Solutions

Analytic FDTD Solutions;

FDTD solutions: superposition of spatial eigen-modes modulated with time varying amplitudes

$$\boxed{[E]^n = \sum_i \alpha_i^n [V]_i}$$

$$\boxed{\alpha_i^{n+1} + \alpha_i^{n-1} - (2-\gamma_i)\alpha_i^n = b_i^n, \ \text{for } n = 1, 2, 3, \ldots}$$

$$\alpha_i^n = \sum_{k=1}^{M} b_i^k \frac{\sin(n-k)\theta_i}{\sin\theta_i} u(n-k)$$

Eigenvalue

$$\theta_i = \arctan\left(\frac{\sqrt{4\gamma_i - \gamma_i^2}}{2-\gamma_i}\right).$$

Source coefficient

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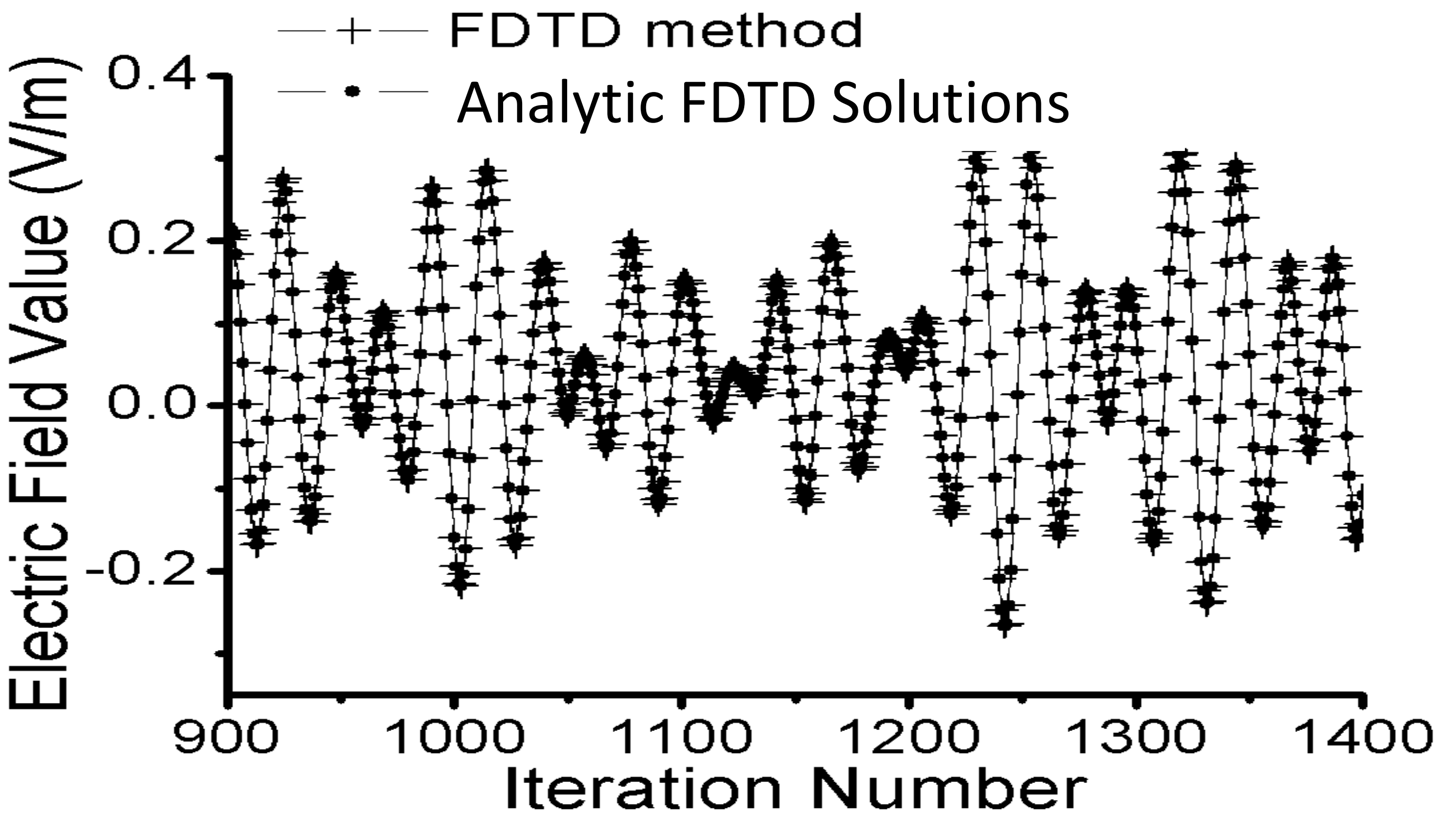
Analytical FDTD Solutions

Analytic FDTD Solutions:



W. Fan, Z. Chen, and S. Yang, “On the analytical solution of the FDTD,” *IEEE Transactions on Microwave Theory and Techniques*, Vol.64, No. 11, Nov. 2016, pp.3370-3379.

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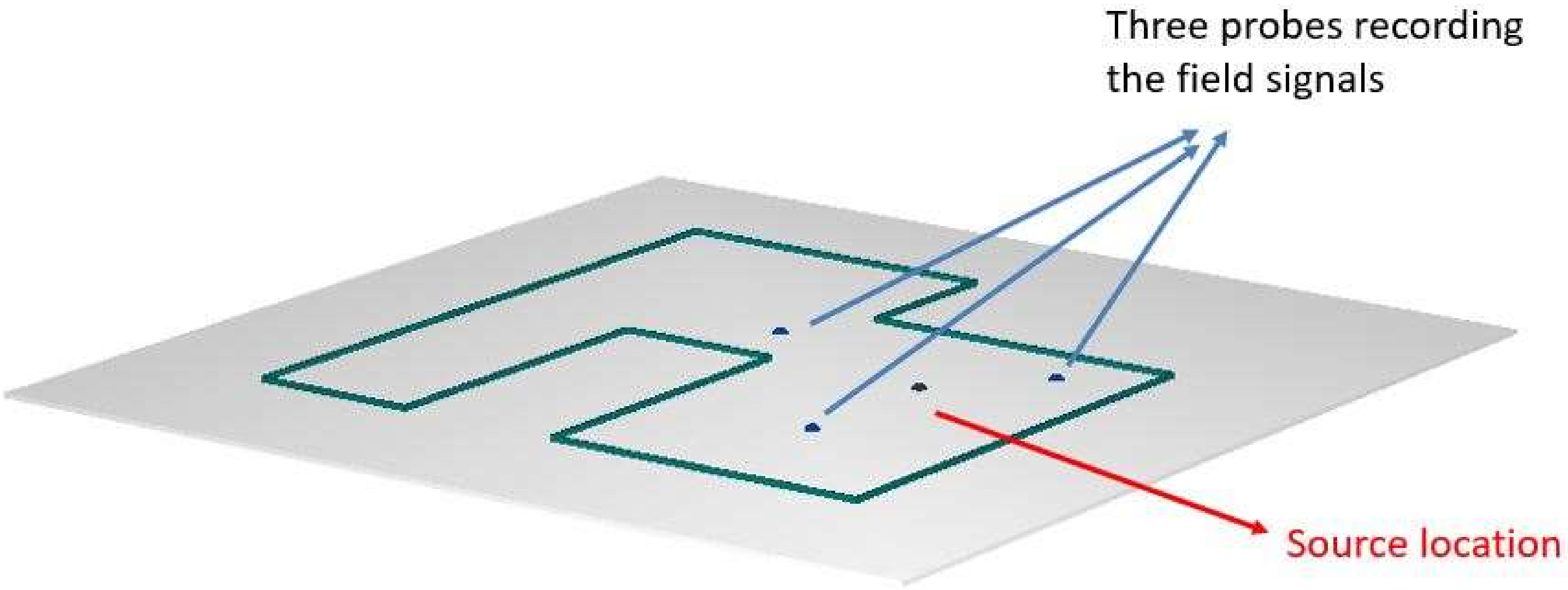
A Time Reversal Method

Time-reversal and structure synthesis

Normal forward wave propagation:



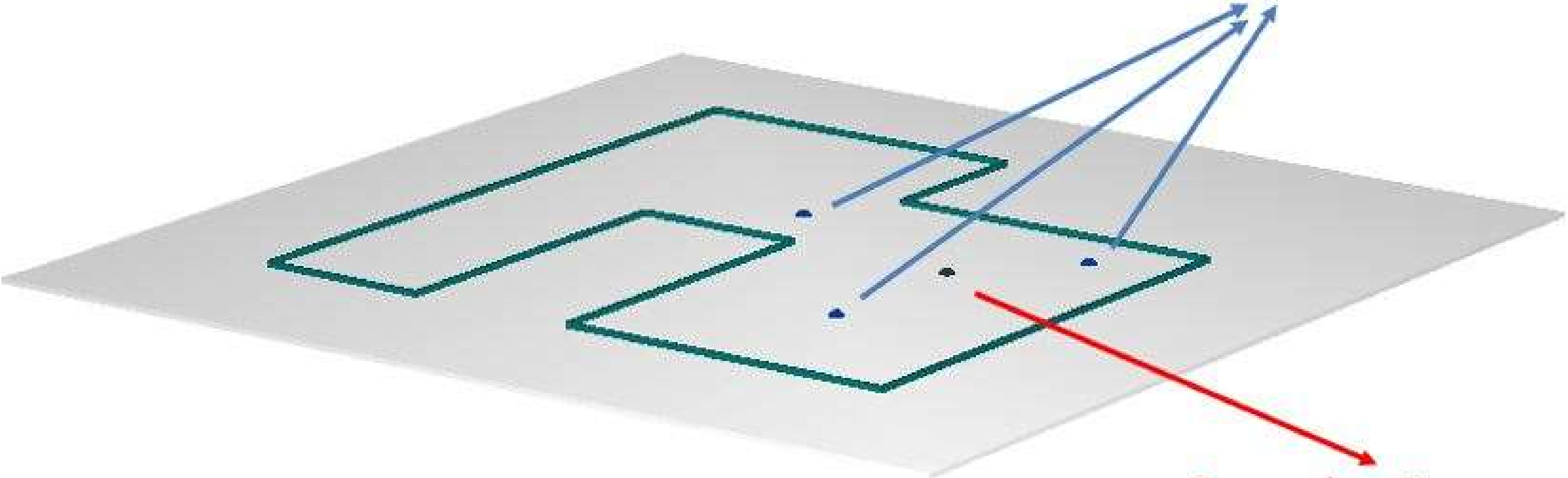
Three probes recording the field signals

Source location

A Time Reversal Method

Time-reversal and structure synthesis:

Reverse (backward) propagation:



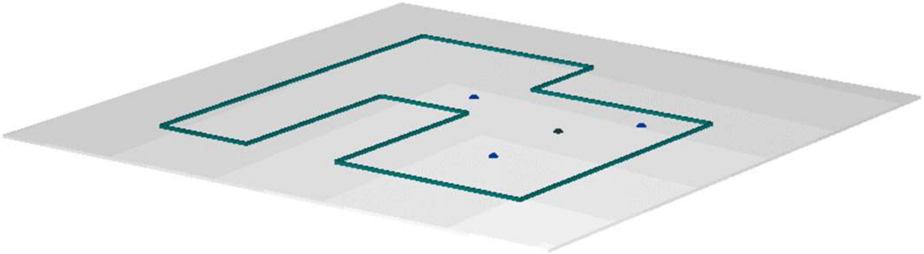
The probes take the recorded signals,
time reverse them and reject them
into the regions → backward simulations

Source location recovered

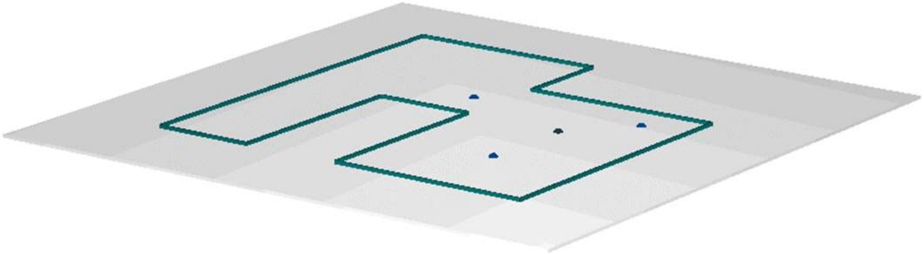
A Time Reversal Method

Time-reversal and structure synthesis:

Simulation



Forward Simulation

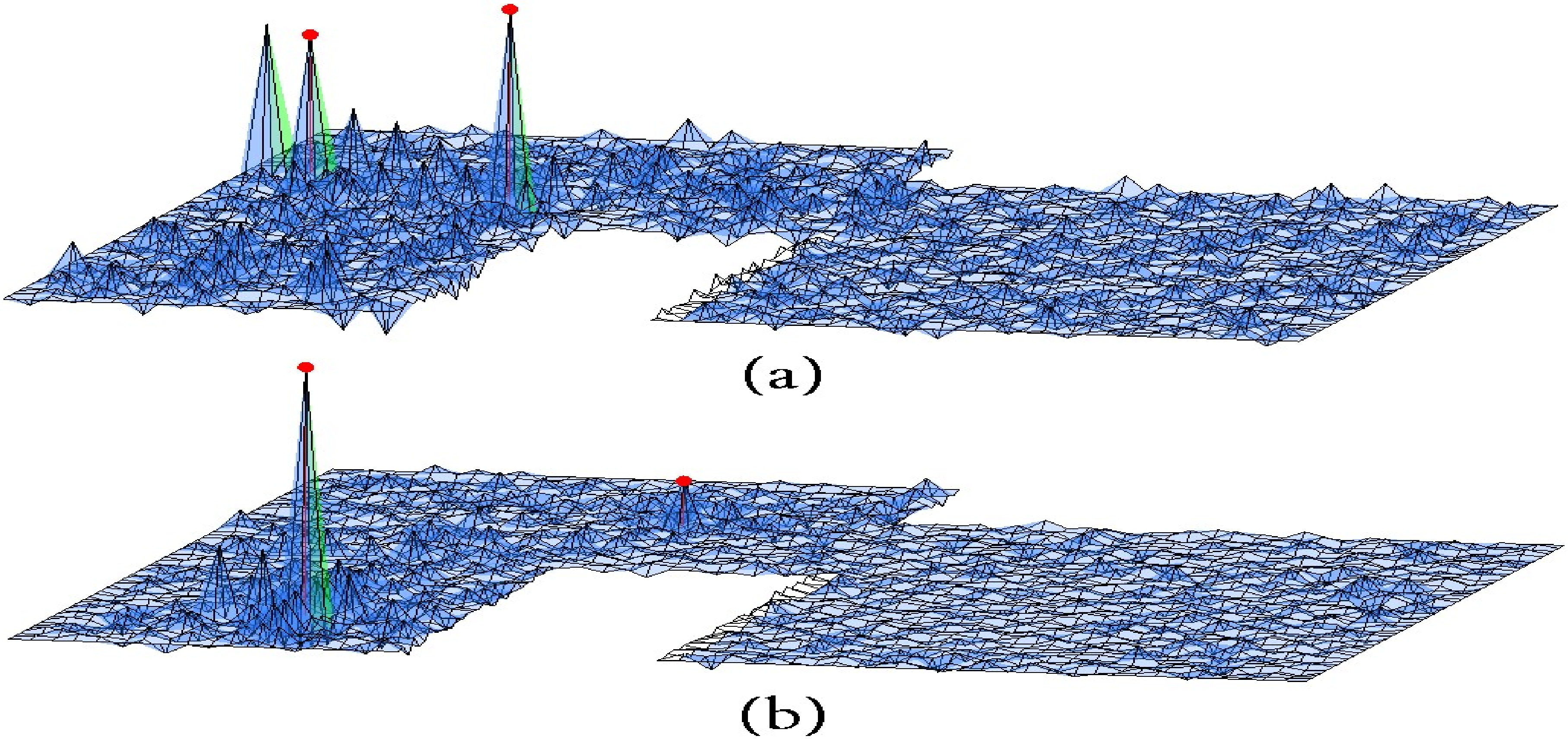


Backward Simulation

A Time Reversal Method

Time-reversal and structure synthesis: different source locations

Failure



A Time Reversal Method

Time-reversal and structure synthesis: the proposed formulation as an optimization problem - regularized least square (RLS) formulation

$$\text{minimize}\ \left(\left\| [S][a] - \left[\sum_{n=1}^{N} y_j{}^2[n] \right] \right\|^2 + \lambda |a| \right)$$

Proposed backward simulation responses

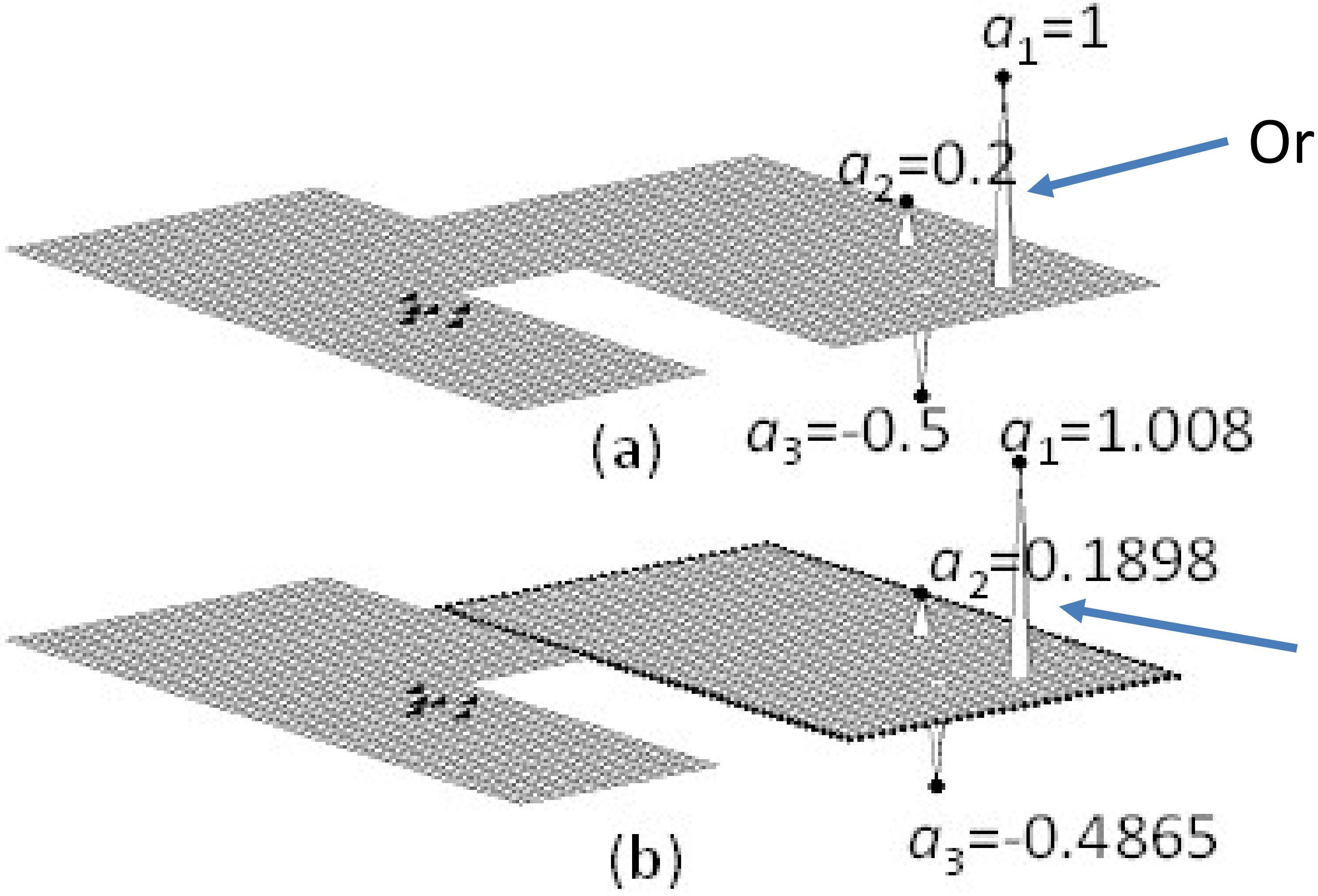
Source amplitude

Forward simulation responses

regularization factor

A Time Reversal Method

Time-reversal and structure synthesis: the proposed optimization formulation → results with impulse responses: full frequency band output signals

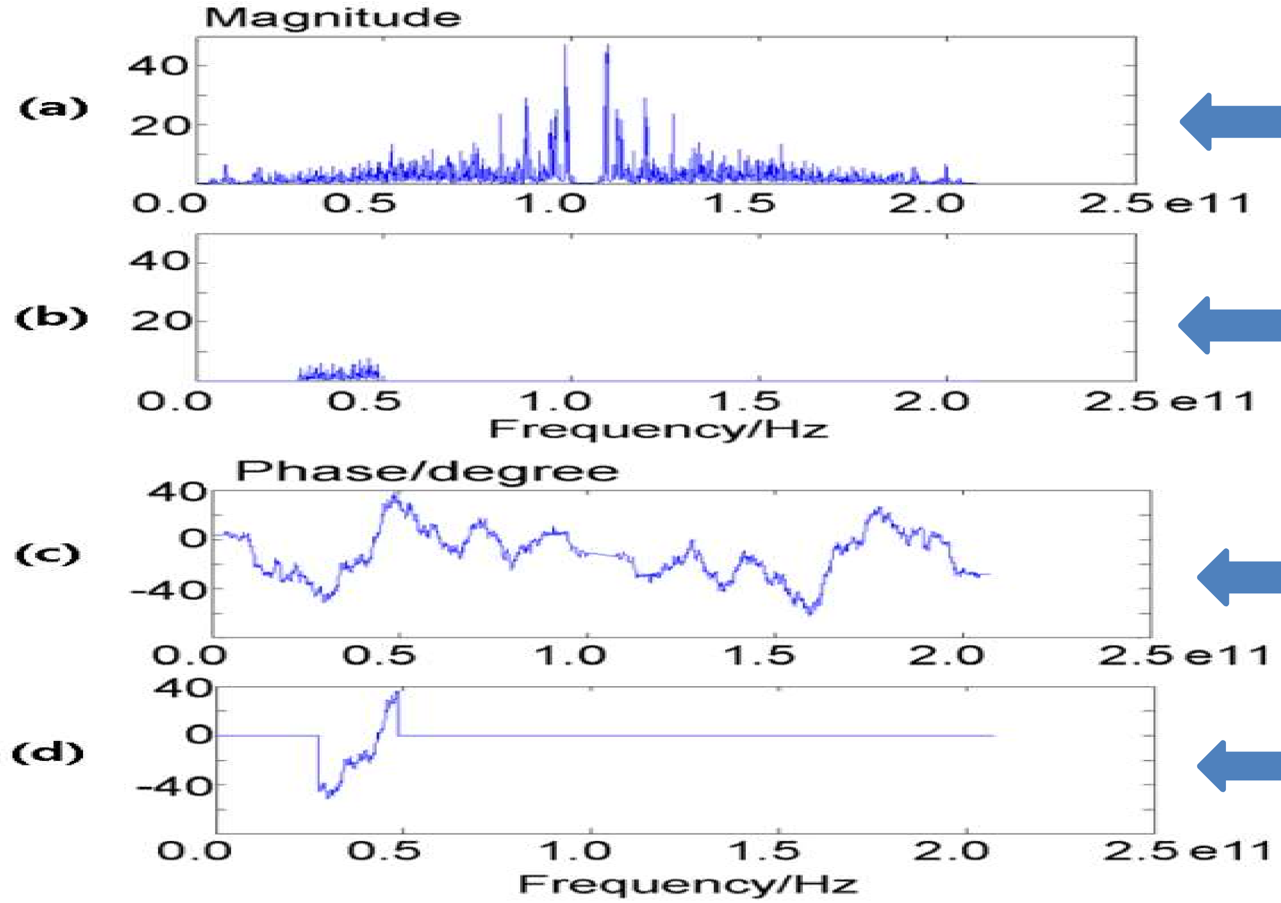


Original sources

Reconstructed or synthesized sources

A Time Reversal Method

Time-reversal and structure synthesis: optimization formulation; in practice: band-limited output signals



Full-frequency-band: magnitude

Limited-frequency-band: magnitude: practical signal

Full-frequency-band: phase

Limited-frequency-band: phase: practical signal

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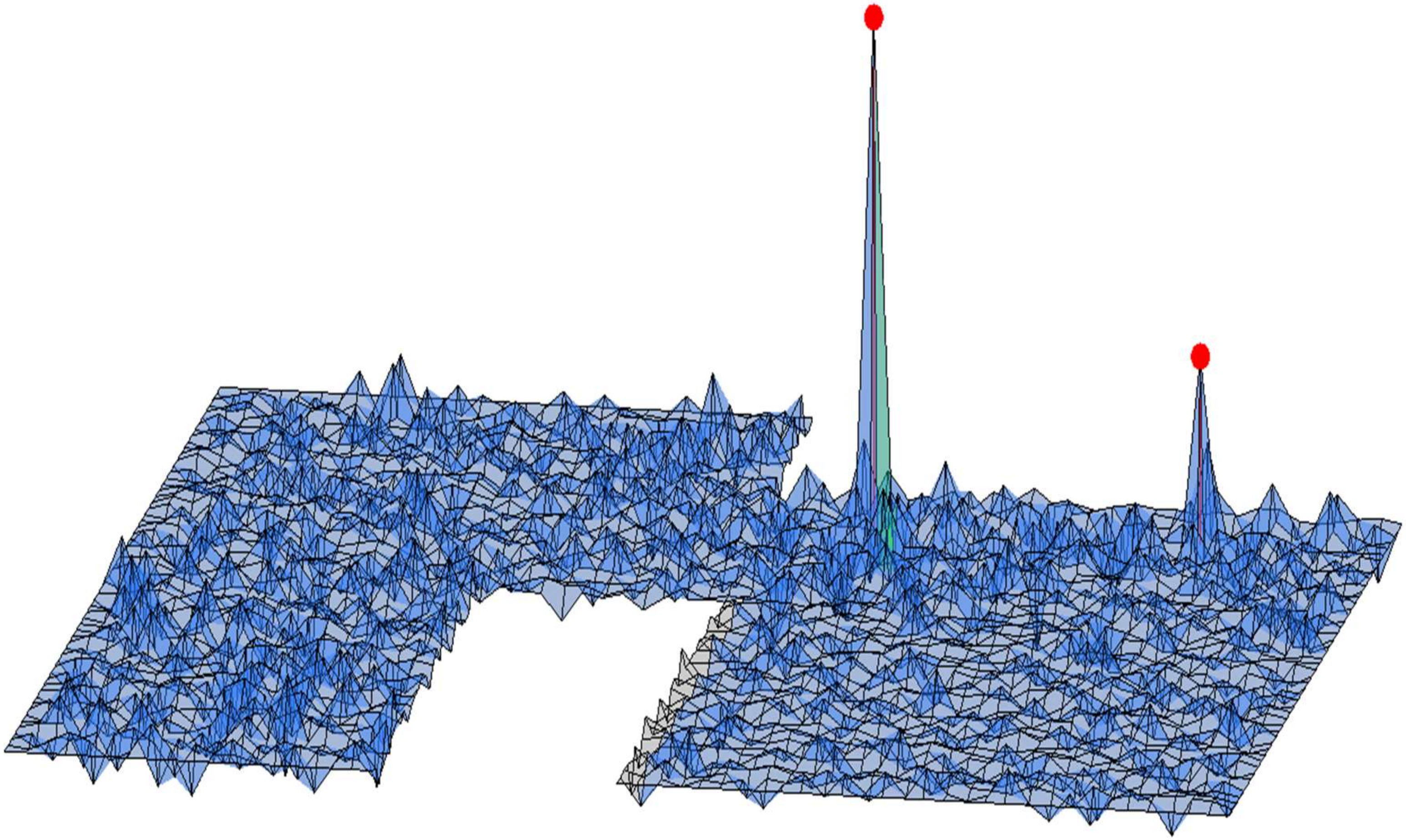
A Time Reversal Method

Time-reversal and structure synthesis: **optimization formulation; band-limited output signals; results.**



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A Time Reversal Method

Time-reversal and structure synthesis: references.

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- R. Sorrentino, L. Rosseli, and P. Mezzanote, “Time-reversal in finite difference time domain method”, *IEEE Microw. Guided Wave Lett.*, vol. 3–11, no. 11, pp. 402–404, Nov. 1993.
- W. Fan and Z. Chen, "A condition for multiple source reconstructions with the time-reversal methods," in *Digest 2016 IEEE MTT-S International Microwave Symposium (IMS)*, San Francisco, CA, 2016, pp. 1-4.

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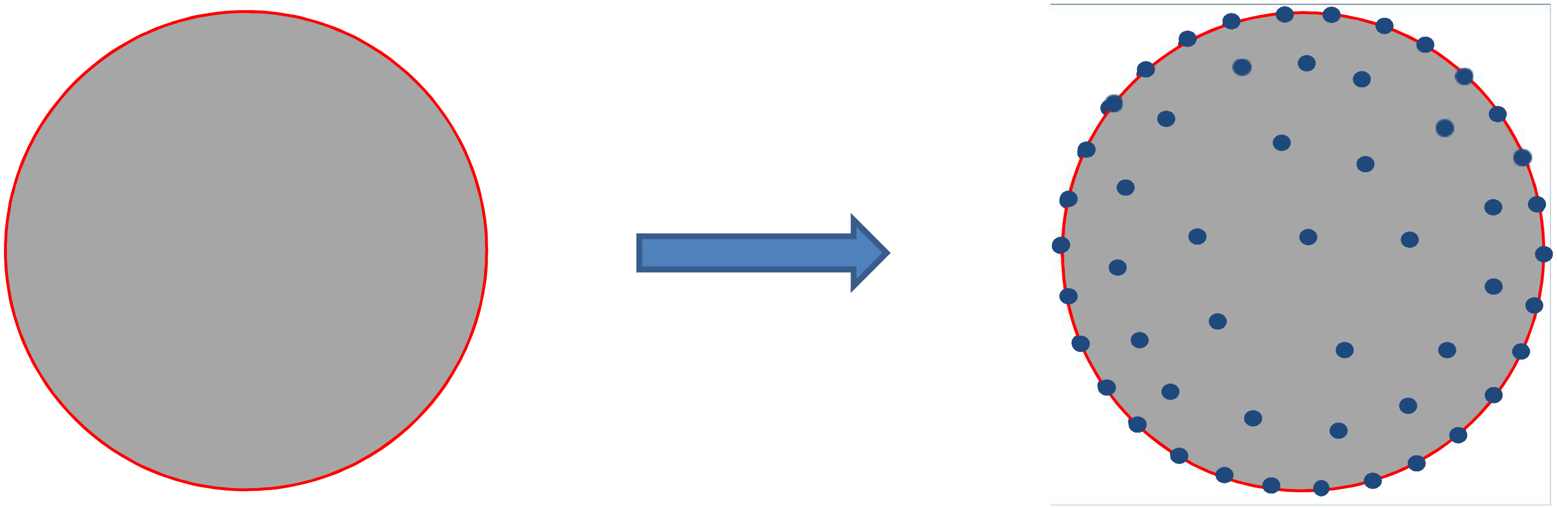
A Unifying Computational Platform

A generalization theory and a unifying computational platform:



Node-based Meshless

Circular

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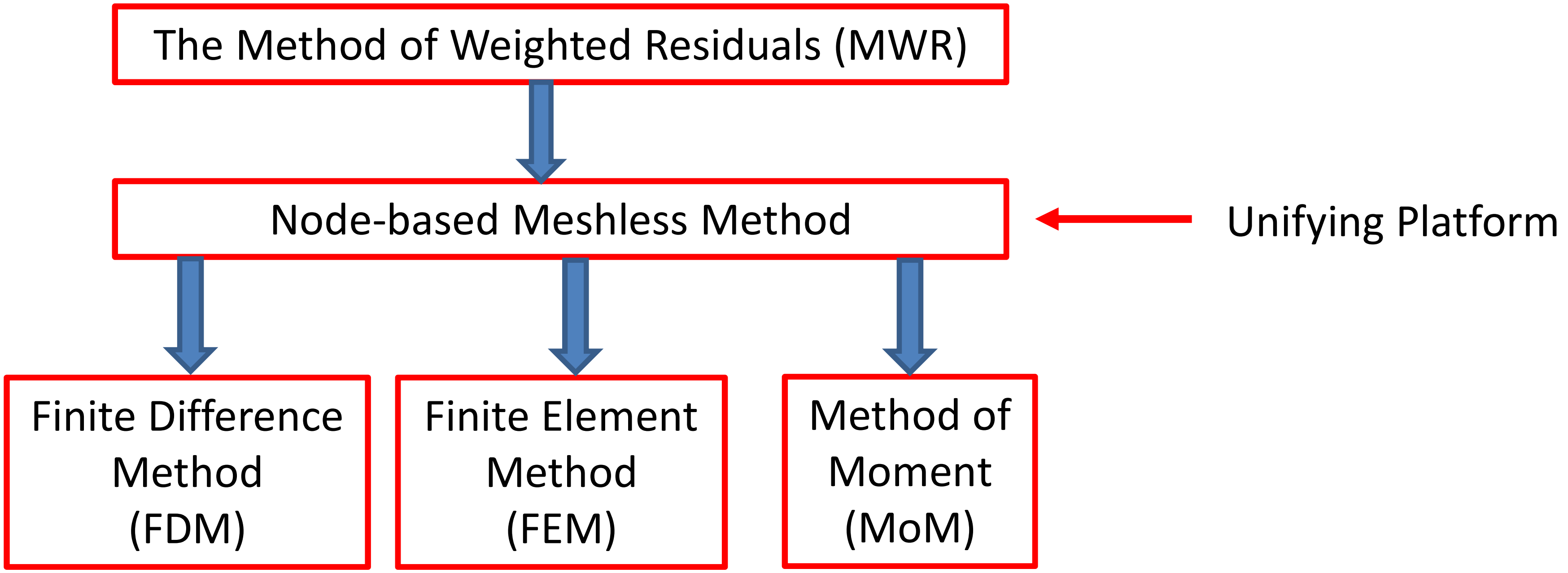
A Unifying Computational Platform

A generalization theory and a unifying computational platform:



The Method of Weighted Residuals (MWR)

Node-based Meshless Method

Unifying Platform

Finite Difference Method (FDM)

Finite Element Method (FEM)

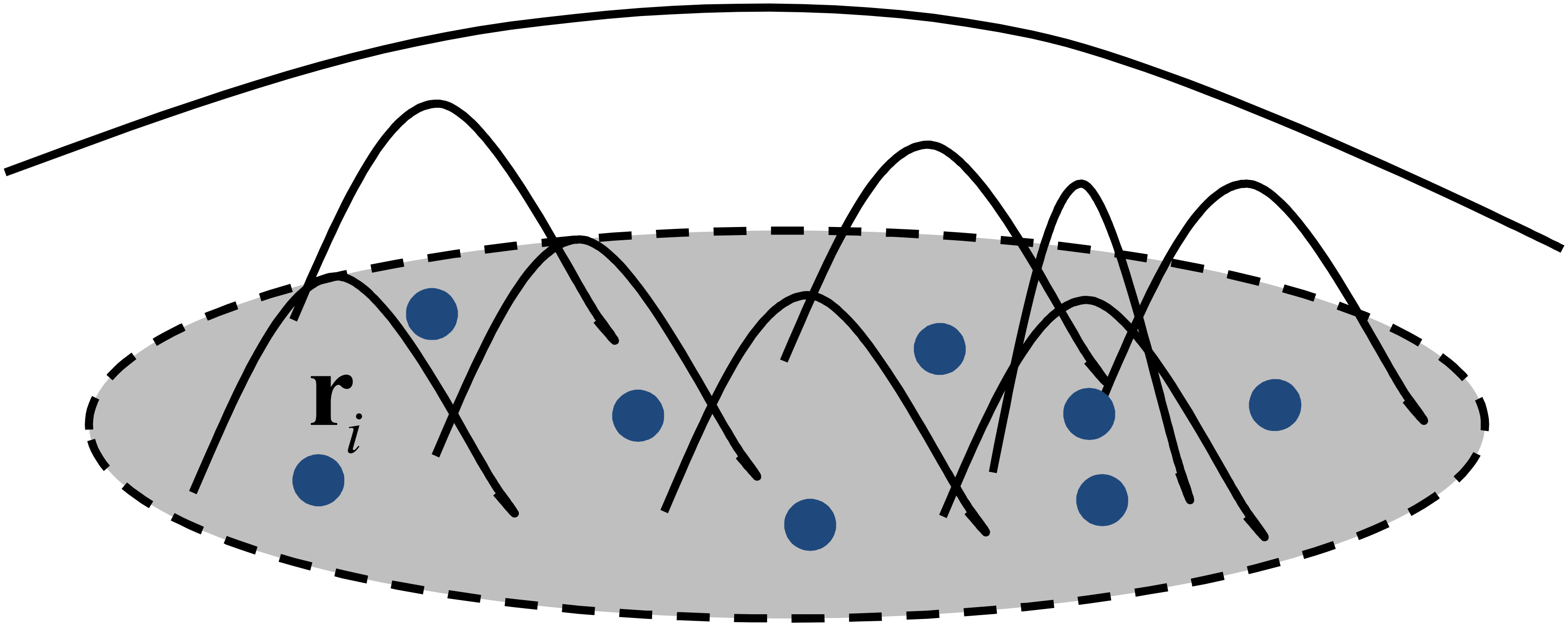
Method of Moment (MoM)

A Unifying Computational Platform

A generalization theory and a unifying computational platform:

Node-based Meshless

$$E(\mathbf{r},t) \text{ or } H(\mathbf{r},t) = \sum_{i=1}^{n} \phi(\mathbf{r},\mathbf{r}_i) a_i(t)$$

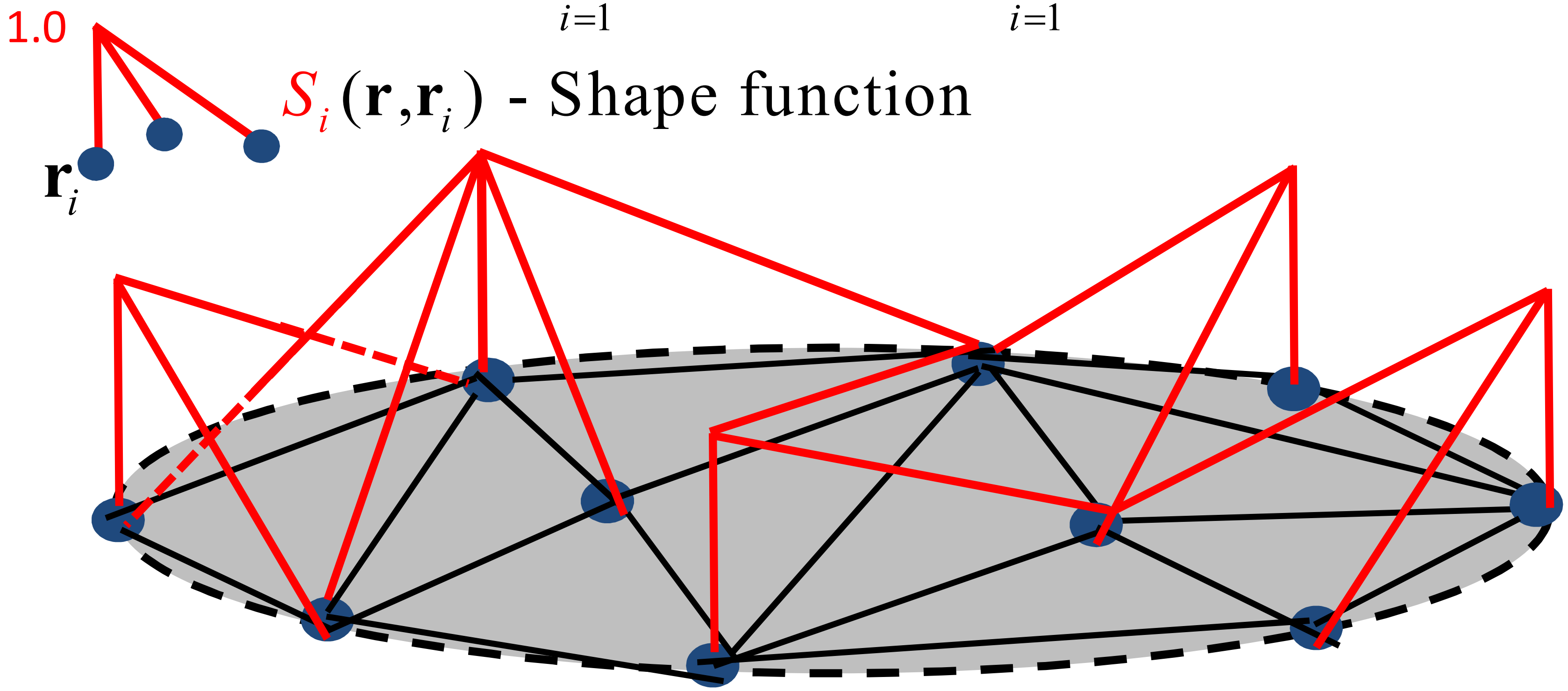


A Unifying Computational Platform

A unifying platform:

Node-based Meshless → Special case: Finite Element Method (FEM)

$$E(\mathbf{r},t) \text{ or } H(\mathbf{r},t)=\sum_{i=1}^{n}\phi(\mathbf{r},\mathbf{r}_i)a_i(t)=\sum_{i=1}^{n}S_i(\mathbf{r},\mathbf{r}_i)a_i(t)$$



$S_i(\mathbf{r},\mathbf{r}_i)$ - Shape function

Node positions are determined by triangular grids

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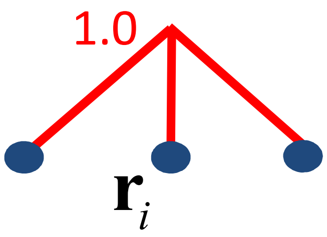
A Unifying Computational Platform

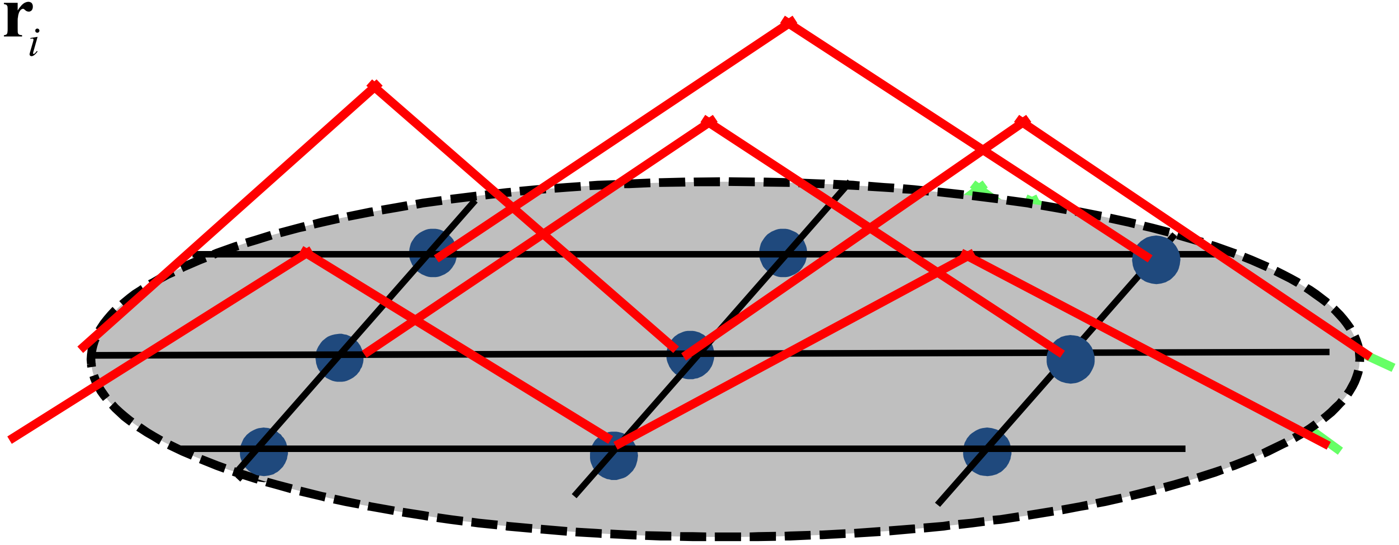
A unifying platform:

Node-based Meshless → Special case: FEM → Special Case: FDTD

$$E(\mathbf{r},t) \text{ or } H(\mathbf{r},t) = \sum_{p=1}^{P} \phi(\mathbf{r},\mathbf{r}_p)a_p(t) = \sum_{i.j.k,} T_i(x,x_i)T_j(y,y_j)T_k(z,z_k)a_{i,j,k}(t)$$



$T_i(r,r_i)$ - Triangular or rooftop function



Node positions are determined by rectangular grids

A Unifying Computational Platform

A unifying platform:

Node-based Meshless → Special case: FEM → Special Case: FDTD

1. Z. Chen and M. Ney, “**Method of weighted residuals: a general approach to deriving time- and frequency-domain numerical methods**,” *IEEE Antennas and Propagation Magazines*, February, 2009 (a feature article), pp.51-70
2. Z. Chen and S. Luo, “**Generalization of the finite-difference based time-domain methods using the method of moments**,” *IEEE Trans. on Antennas and Propagation*, Vol. 54, No. 9, September, 2006, pp. 2515-2524
3. Z. Chen and Y. Yu, “**Node-based methods: a unifying approach to developing numerical techniques**,” invited special session paper, in *Proc. of Intl. Conf. on Electromagnetics in Advanced Applications*, Sept. 12-16, 2011, Torino, pp.737-740
4. J. Wang, Z. Chen, et. al., “**A computational platform that unifies and hybridizes numerical methods**,” in *Digest 2017 2017 IEEE International Symposium on Antennas and Propagation*, July 9-14, 2017, San Diego

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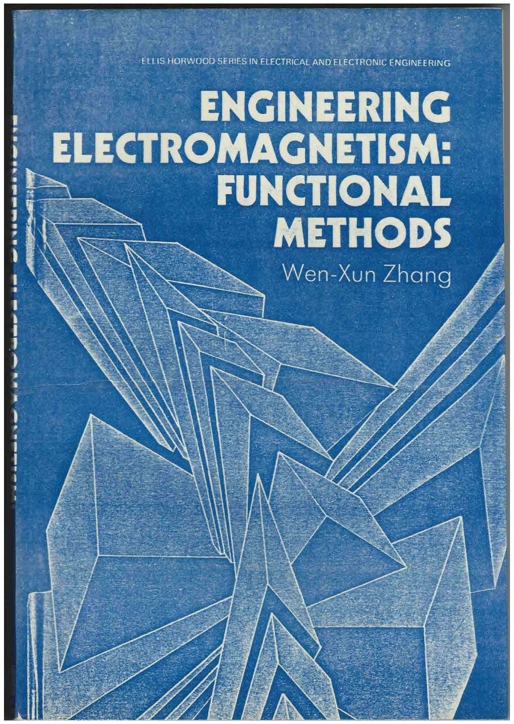
A Unifying Computational Platform

A very good but unfortunately not-well known early manograph by Prof. Wen-xun Zhang implies some of the ideas of the unification of the different numerical methods


ELLIS HORWOOD SERIES IN ELECTRICAL AND ELECTRONIC ENGINEERING
ENGINEERING ELECTROMAGNETISM: FUNCTIONAL METHODS
Wen-Xun Zhang

W. X. Zhang, *Engineering Electromagnetism: Functional Method*, Ellis Norwood, 1991

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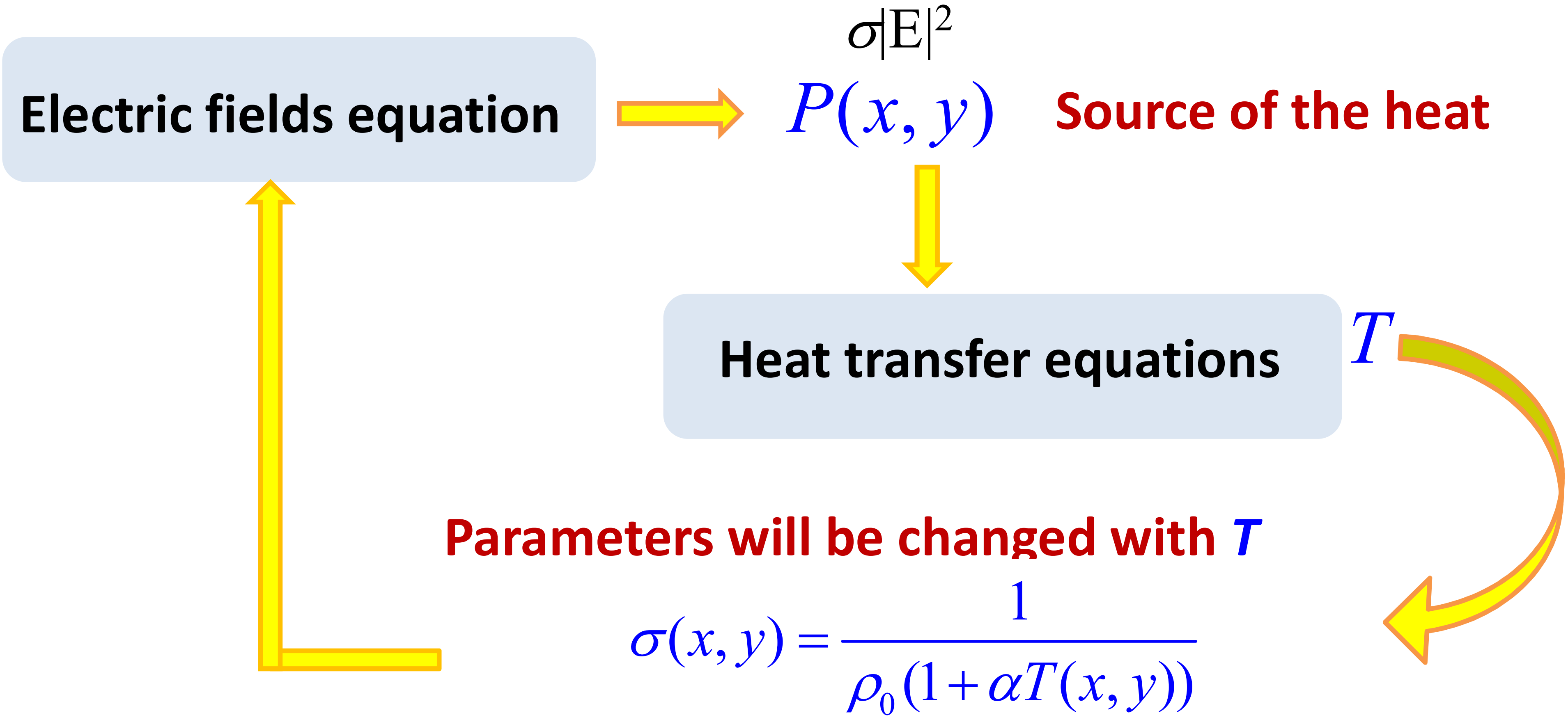
A Unifying Computational Platform

Multiphysics and multiscale modeling

Example: electric fields equation interact with the thermal equation?



Electric fields equation

$\sigma|\mathrm{E}|^2$

$P(x, y)$ **Source of the heat**

Heat transfer equations

T

Parameters will be changed with *T*

$$\sigma(x, y) = \frac{1}{\rho_0(1+\alpha T(x, y))}$$

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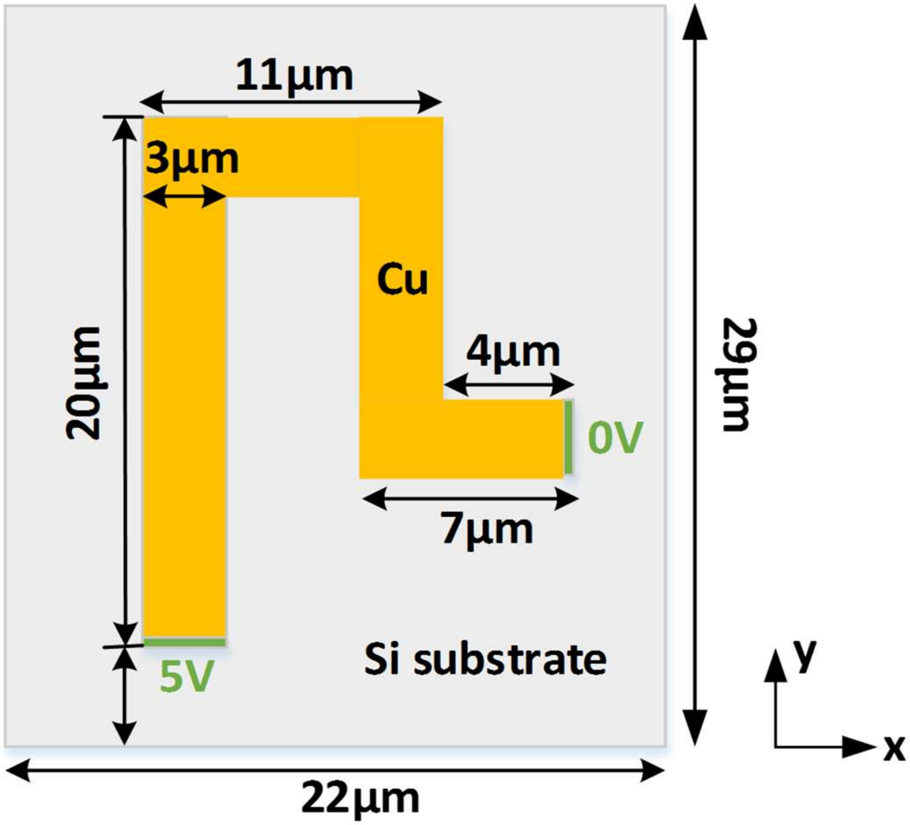
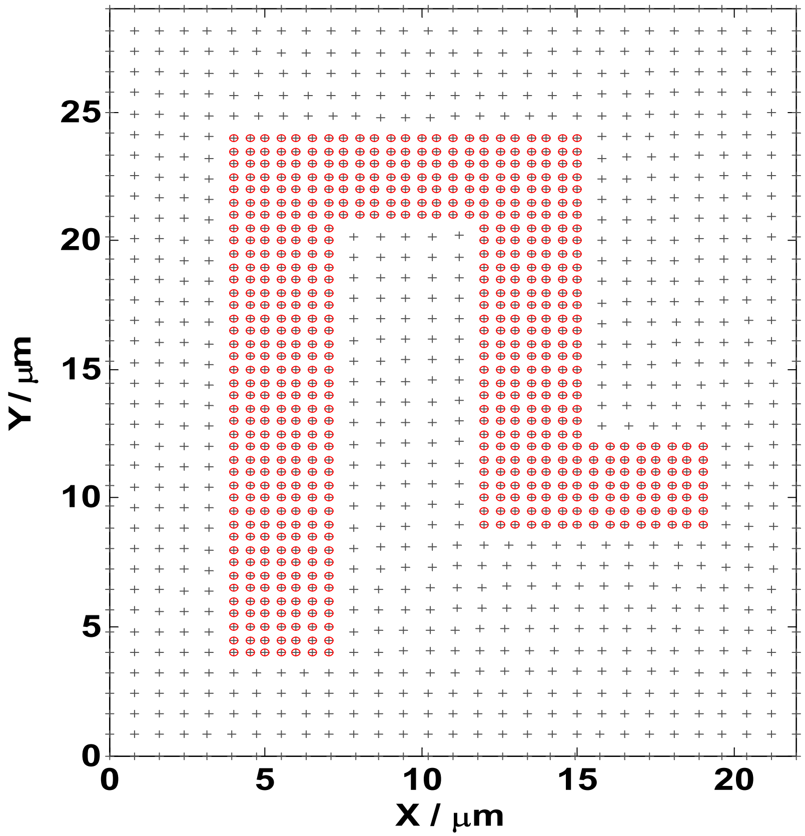
A Unifying Computational Platform

Multiphysics and multiscale modeling


11µm
3µm
Cu
4µm
0V
20µm
7µm
29µm
5V
Si substrate
22µm
y
x



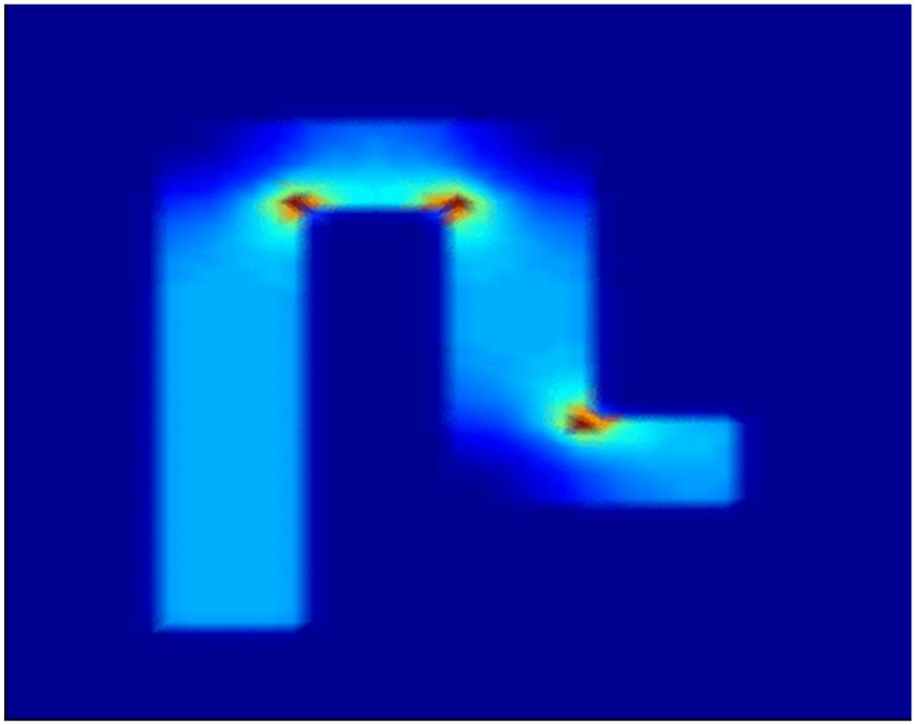
Multiscale nodes distribution

X. Zhang, Z. Chen, and Y. Yu, “**An Unconditional stable meshless ADI-RPIM for simulation of coupled transient electro-thermal problems**,”, *IEEE Journal on Multiscale and Multiphysics Computational Techniques*, Oct. 31, 2016, pp. 98-106.

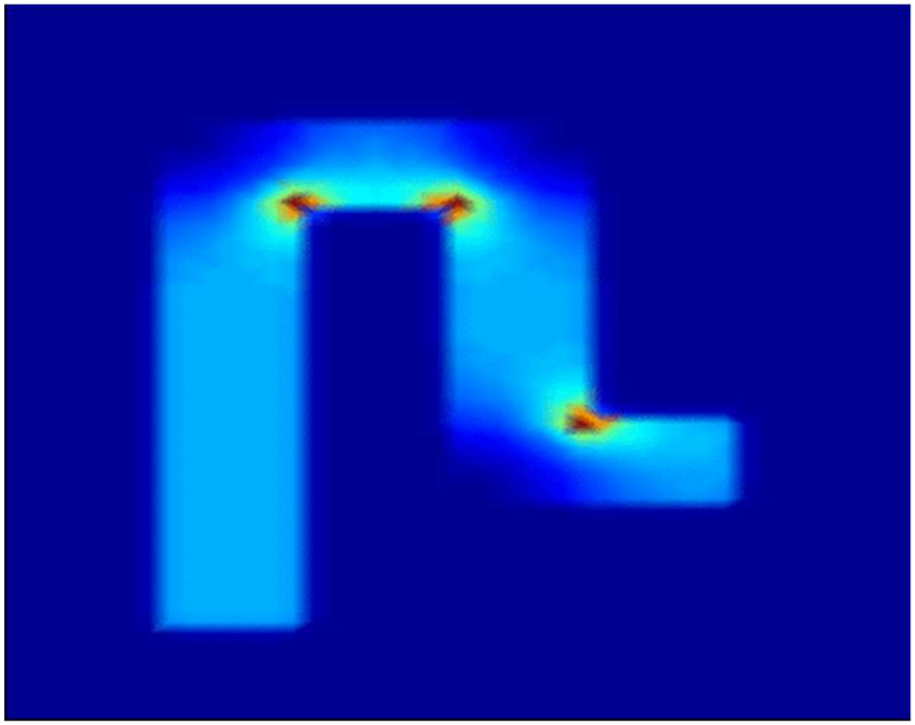
A Unifying Computational Platform

Multiphysics and multiscale modeling

Simulation



Dirichlet boundary condition



Robin boundary condition

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Conclusion

Eigen-mode and Analytic Representation of Numerical Solutions

- The FDTD solutions are found to be the superposition of the spatial eigen modes.
- How to deal with these spatial eigenmodes may hold the key for further improving the FDTD computational efficiency.
- Analytic FDTD solutions have been found, promising for efficient time-domain modeling.
- Similar can be applied to other time-domain methods.
- Non-conformal or unstructured methods are naturally effective for multiscale modeling.

Conclusion

The Proposed Time Reversal Method

- Time reversal techniques may be an alternative way to synthesize the sources or structures if the forward simulation results can be specified as the desired responses.
- A optimization formulation with the time reversal method has been proposed as an effective way to reconstruct sources without spurious results.
- The proposed method works in the case of bandlimited signals at the output nodes, making the method practical.

Conclusion

A Unifying Computational Platform

- There is not a single numerical method that is good at solving all the electromagnetic problems (e.g. multiscale or Multiphysics) → interfacing or hybridization of different methods may solve them all → a unifying platform is then desirable.
- The node-based mesheless method can be used as the unifying platform although its theory and applications have not been fully established yet.

Congratulations and Happy Birth to Prof. Wen-Xun Zhang!

HAPPY BIRTHDAY